

Continuous quantum measurement: inelastic tunnelling suppresses current oscillations

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We study the dynamics of a charge qubit, consisting of a single electron in a double well potential coupled to a point-contact (PC) electrometer, using the quantum trajectories formalism. Contrary to previous predictions, we show formally that, in the sub-Zeno regime, coherent oscillations in the detector output are suppressed by inelastic processes in the PC. Furthermore, these reduce the detector efficiency. We show that the sub-Zeno dynamics are divided into two regimes: low- and high-bias in which the PC current power spectra show markedly different behaviour.

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Single shot quantum measurement of mesoscopic systems is recognised as an important goal. Fundamentally, it will allow us to make time-resolved observations of quantum mechanical effects in such systems, and practically it will be a necessary component in the construction of solid-state quantum information processors (QIP). There are numerous proposals for implementing QIP in solid state systems, using doped silicon [1], electrostatically defined quantum dots [2] and superconducting boxes [3]. In these proposals, the output of the QIP is determined by measuring the position of a single electron or Cooper pair. Single electrons hopping onto single quantum dots have been observed on a microsecond time scale using single-electron transistors (SET) [4]. Ensemble measurements of a double well system (qubit) have been demonstrated in superconducting devices [5]. So far, single shot qubit measurements remain elusive.

It is therefore important to consider the measurement of single electron qubits by sensitive electrometers. Two possible electrometers have been discussed to date: SETs (see e.g. [3]) and point contacts (PCs) [6, 7, 8, 9, 10, 11, 12, 13, 14]. PCs have been shown to be sensitive charge detectors [15, 16, 17, 18], and are the focus of this Letter. Figure 1 illustrates the physical system we consider here, with a PC (represented by two Fermi seas separated by a tunnel barrier) in close proximity to one of the double well minima. Despite the apparent simplicity of this system it exhibits a rich range behaviour.

There are three energy scales of relevance: the splitting of the qubit eigenstates, ϕ , the PC bias voltage, eV , and the measurement induced dephasing rate, $\Gamma_d = 2(\sqrt{\Gamma_l} - \sqrt{\Gamma_r})^2/e$, due to the distinct currents, $I_{l,r}$, through the PC when the qubit electron is held in the left (l) or right (r) well. There may also be environmental decoherence [13], which we ignore. The three energies define three distinct measurement regimes: (1) low-bias regime, where $\Gamma_d/2 \ll eV < \phi$, (2) high-bias regime, where $\Gamma_d/2 \ll \phi < eV$ and (3) quantum Zeno limit, where $\phi \ll \Gamma_d/2 \ll eV$.

In the quantum Zeno limit, frequent weak measurements localise the qubit, suppressing its dynamics [6, 10].

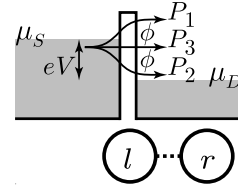


FIG. 1: Schematic of the qubit and PC showing lead energy bands. Electrons tunnelling from the source to the drain may do so elastically or inelastically, depicted by arrows. Different transitions induce different jumps, P_i , on the qubit.

To date, analyses have not distinguished between the low- and high- bias regimes, and it has been predicted that in the limit, $\Gamma_d/2 \ll \phi$, which we refer to as the sub-Zeno limit, coherent oscillations due to the quantum dynamics of the qubit will be observed in the power spectrum of the current through the PC [6, 9, 10, 13]. It has also been claimed that the PC is an efficient detector, that is, no information about the qubit is lost by the detector. We present a detailed analysis of the sub-Zeno limit, showing the sharp difference in behaviour between the low- and high- bias regimes, and we show that previously predicted coherent oscillations are suppressed by inelastic tunnelling of electrons through the PC. The analysis also formally yields the boundary at which approximations leading to the Zeno effect are reasonable.

For a low transparency PC, the Hamiltonians, in units where $\hbar = 1$, for the qubit, leads, and their interaction are given by [6, 7, 8, 9, 10, 13, 14]

$$H_{\text{sys}} = (-\Delta \sigma_x - \epsilon \sigma_z)/2 = \phi \sigma_z^{(e)}/2, \quad (1)$$

$$H_{\text{meas}} = \sum_{k,q} (T_{k,q} + \chi_{k,q} \sigma_z) a_{D,q}^\dagger a_{S,k} + \text{H.c.} \quad (2)$$

$$H_{\text{leads}} = H_S + H_D = \sum_k \omega_k (a_{S,k}^\dagger a_{S,k} + a_{D,k}^\dagger a_{D,k}) \quad (3)$$

where $\sigma_x = |l\rangle\langle r| + |r\rangle\langle l|$, $\sigma_z = |l\rangle\langle l| - |r\rangle\langle r|$, $\sigma_z^{(e)} = |e\rangle\langle e| - |g\rangle\langle g|$, $\theta = \tan^{-1}(\frac{\Delta}{\epsilon})$, $\phi = \sqrt{\Delta^2 + \epsilon^2}$ and $|e\rangle = -\sin(\theta/2)|l\rangle + \cos(\theta/2)|r\rangle$. We will adopt the convention that $\chi_{k,q} < 0$ so that the left well is nearest the PC.

Unconditional master equation. We derive a master equation for the reduced density matrix of the qubit based on the open quantum systems approach described in [19]. The von Neumann equation for the density matrix, R , of the bath and system is $\dot{R}(t) = -i[H_{\text{Tot}}, R]$. To derive the master equation for the reduced density operator, ρ , of the double well system we transform to an interaction picture with respect to the free Hamiltonian $H_0 = H_{\text{sys}} + H_{\text{leads}}$, $H_I(t) = e^{iH_0 t} H_{\text{Tot}} e^{-iH_0 t} - H_0 = e^{iH_0 t} H_{\text{meas}} e^{-iH_0 t}$. We formally integrate the von Neumann equation, then trace over lead modes,

$$\dot{\rho}_I(t) = \text{Tr}_{S,D} \left\{ -i[H_I(t), R(0)] - [H_I(t), \int_0^t dt' [H_I(t'), R(t')]] \right\}, \quad (4)$$

where $H_I(t) = \sum_{k,q} S_{k,q}(t) a_{D,q}^\dagger a_{S,k} + \text{H.c.}$, and $S_{k,q}(t) = e^{i(\omega_k - \omega_q)t} (T_{k,q} + \chi_{k,q} \sigma_z(t))$. We write $S_{k,q}(t)$ as a discrete Fourier decomposition $S_{k,q}(t) = e^{-it(\omega_k - \omega_q)} (e^{-it\phi} P_1 + e^{it\phi} P_2 + P_3)$, where $P_1 = P_2^\dagger = \frac{\chi_{00} \sin(\theta)}{2} |g\rangle\langle e|$, $P_3 = (T_{00} + \frac{\chi_{00} \cos(\theta)}{2} \sigma_z^{(e)})$. We have assumed that $T_{k,q} = T_{00}$ and $\chi_{k,q} = \chi_{00}$ are constant. Furthermore the PC is weakly responding so $\chi_{00} \ll T_{00}$.

The form of $S_{k,q}(t)$ indicates that there are three possible jump processes, indicated in Fig. 1. P_3 is associated with elastic tunnelling of electrons through the PC. P_1 (P_2) is associated with inelastic excitation (relaxation) of electrons tunnelling through the PC with an energy transfer ϕ . This energy is provided by the qubit which relaxes (excites) in response. Inelastic transitions in similar systems have been described in [20], which calculated the current power spectrum through an open double well system due to shot noise through a nearby PC.

We make standard assumptions that result in a time-independent Markovian master equation. Firstly we assume that the leads are always near thermal equilibrium so $\text{Tr}_{S,D} \{ a_{i,k}^\dagger R(t) \} = 0$ and $\text{Tr}_{S,D} \{ a_{i,k}^\dagger a_{n,k} R(t) \} = \delta_{i,j} f_i(\omega_k) \rho(t)$, where $i, j \in \{L, R\}$, f_i is the Fermi distribution for lead i and ρ is the qubit density matrix. Secondly, if the lead correlation time is much less than other time scales, then the lower limit on the integral in Eq. (4) can be set to $-\infty$ [19]. We substitute $\sum_k \rightarrow \int d\omega_k g_{S(D)}$ where $g_{S(D)}$ are the lead density of states, which are assumed constant over the relevant energies.

Finally, we make the rotating wave approximation (RWA), setting to zero terms in the master equation proportional to $e^{\pm i\phi t}$. The RWA is applicable when $\phi \gg \nu^2 eV$ (defining the sub-Zeno limit), where $\nu = \sqrt{\pi g_S g_D / 2 \chi_{00}}$, which is small in the low transparency, weakly responding limit. We also make the approximation $\rho(t') \rightarrow \rho(t)$ in Eq. (4), which is valid when $\nu^2 \ll 1$, to arrive at the interaction picture master equation

$$\dot{\rho}_I(t) = 2\pi g_S g_D \sum_n \left(\mathcal{D}[\sqrt{\Theta(eV + \omega_n)} P_n] \rho_I(t) + \mathcal{D}[\sqrt{\Theta(-eV - \omega_n)} P_n^\dagger] \rho_I(t) \right), \quad (5)$$

where $eV = \mu_S - \mu_D$ is the bias applied across the PC, $\mathcal{D}[B]\rho = \mathcal{J}[B]\rho - \mathcal{A}[B]\rho$, $\mathcal{J}[B]\rho = B\rho B^\dagger$, $\mathcal{A}[B]\rho = \frac{1}{2}(B^\dagger B\rho + \rho B^\dagger B)$, $\Theta(x) = (x + |x|)/2$ is the ramp function and μ_i is the chemical potential of lead i . The first term in Eq. (5) arises from forward tunnelling electrons (source to drain), whilst the second is due to reverse processes. We note that the dynamics implicit in this master equation differs from the dynamics presented previously [6, 7, 8, 9, 10], since it permits inelastic jump process.

In the Zeno limit, $\phi \ll \nu^2 eV$, the RWA breaks down. Instead an alternate approximation is accurate, $e^{i\phi t} \rightarrow 1$, or equivalently $\sigma_z(t) \rightarrow \sigma_z(0)$. In this limit, $S_{k,q}(t) = e^{-i(\omega_k - \omega_q)t} (T_{00} + \chi_{00} \sigma_z(0))$ has only one Fourier component, and we regain the master equation of [6, 8, 9].

Conditional master equation. We wish to provide an unravelling of the master equation into evolution conditioned on measurement results. From the unravelling we can directly calculate the observed current correlation function and the power spectrum. From the unravelling it is also possible to simulate quantum state trajectories consistent with single-shot experimental realisations of measured currents. There is no unique unravelling for the master equation Eq. (5): one may add an arbitrary constant to each of the jump operators [7] in Eq. (5), to produce new (rescaled) operators

$$c_1 = \nu \sqrt{eV + \phi} \sin(\theta) |g\rangle\langle e| + \gamma_1, \quad (6)$$

$$c_2 = \nu \sqrt{|eV - \phi|} \sin(\theta) |e\rangle\langle g| + \gamma_2, \quad (7)$$

$$c_3 = \nu \sqrt{eV} \cos(\theta) \sigma_z^{(e)} + \mathcal{T} \sqrt{eV}, \quad (8)$$

which along with a modification to the system Hamiltonian, provides an identical unconditional master equation, and we have defined $\mathcal{T} = \sqrt{\pi g_S g_D / 2 T_{00}}$.

The unravelling that most accurately represents a given measurement process must be determined from physical considerations. We discuss in detail only the unravelling in the high bias regime, in which all lead electron jumps correspond to tunnelling from the source to the drain, as reverse processes are Pauli blocked. Thus, when measuring currents through the leads, none of the processes are distinguished, since they all just contribute to a current. As a result, when a jump occurs, the resultant state of the qubit is a probabilistic mixture $\tilde{\rho}_{1c}(t + dt) = \sum_n \mathcal{J}[c_n] \rho_c(t) dt$. Between jumps, it evolves smoothly according to $\dot{\rho}_{0c}(t) = -i[H_{\text{sys}}, \tilde{\rho}_{0c}(t)] - \sum_n \mathcal{A}[c_n] \tilde{\rho}_{0c}(t)$.

The constants associated with the inelastic processes, $\gamma_{1,2}$, are eliminated using energy conservation arguments. Imagine we prepare the qubit in the state $|g\rangle$. Then suppose, using a sensitive bolometer measuring the change in energy in the leads, we determined that an inelastic jump had followed. We would conclude that a lead electron tunnelling from the source to drain lost an amount of energy $-\omega_2 = \phi$. Simultaneously, the qubit evolves discontinuously, $|g\rangle \rightarrow c_2 |g\rangle$. If $\gamma_2 = 0$ then this state is $|e\rangle$, so the qubit gains an energy ϕ , and energy is conserved, as required. Otherwise, if $\gamma_2 \neq 0$ then it is straightforward

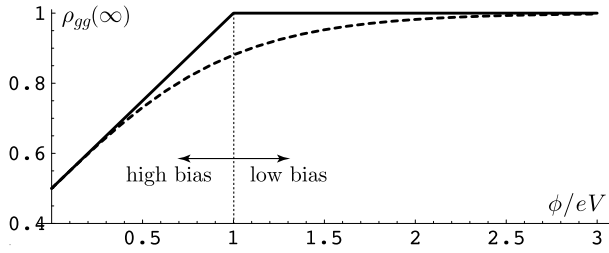


FIG. 2: Equilibrium ground state occupation probability for a qubit near a PC (solid), and for a thermalised qubit, $\rho_{gg}^{\text{therm}}(\infty)$, at a temperature $T = eV/2$ (dashed).

to show that energy is not conserved on average. A similar argument gives $\gamma_1 = 0$, and these arguments also hold in the low bias regime. Though lengthier, the same reasoning applies when considering current measurements (rather than bolometric), resulting in $\gamma_1 = \gamma_2 = 0$.

In order to determine the final constant in the unravelling, \mathcal{T} , we relate the current through the PC to the jump processes according to the relation

$$I(t)dt = e \text{Tr} \{ \tilde{\rho}_{1c}(t + dt) \} = e \sum_n \text{Tr} \{ \mathcal{J}[c_n] \rho_c(t) \} dt. \quad (9)$$

In the absence of tunnelling ($\theta = 0$), if the qubit is localised in the energy eigenstate $|l\rangle$ ($|r\rangle$) we expect a current $I_l \equiv eT_l^2$ ($I_r \equiv eT_r^2$) to flow through the PC. Solving the two equations $\sum_n \text{Tr} \{ c_n |l\rangle \langle l| c_n^\dagger \} = eV(\mathcal{T} - \nu)^2 = T_l^2$ and $\sum_n \text{Tr} \{ c_n |r\rangle \langle r| c_n^\dagger \} = eV(\mathcal{T} + \nu)^2 = T_r^2$ yields $\mathcal{T} = (T_r + T_l)/2\sqrt{eV}$ and $\nu = (T_r - T_l)/2\sqrt{eV}$. The average and difference in the currents are $\bar{I}_{\text{loc}} = (I_r + I_l)/2 = e(\mathcal{T}^2 + \nu^2)eV$ and $\delta I_{\text{loc}} = I_r - I_l = -4e\nu\mathcal{T}eV$.

For general values of θ the mean current does not change, $\bar{I}(\theta) = \bar{I}_{\text{loc}}$, but the difference in current between the two localised states is $\delta I(\theta) = e\nu\mathcal{T} \cos^2(\theta) eV$, to lowest order in ν . Notably, the variation in current between localised states vanishes when $\theta = \pi/2$. This demonstrates that when the energy eigenstates are completely delocalised, the PC measurement does not localise the qubit. This is evident from the form of the jump operators, since when $\theta = \pi/2$, c_3 does not affect the qubit whilst $c_{1,2}$ only induce transitions between the symmetric ($|g\rangle$) and antisymmetric ($|e\rangle$) states.

Results. In the high bias regime, the master equation is given by Eq. (5) with $\Theta(-eV - \omega_n) = 0$. Solving the high-bias master equation in the interaction picture gives

$$\begin{aligned} \rho_{ge}(t) &= e^{-\Gamma_d(1+\cos^2(\theta))t} \rho_{ge}(0), \\ \rho_{gg}(t) &= \frac{\phi + eV}{2eV} - e^{-\Gamma_d \sin^2(\theta)t} \left(\frac{\phi + eV}{2eV} - \rho_{gg}(0) \right), \end{aligned} \quad (10)$$

where we have defined $\Gamma_d = 2\nu^2 eV$.

Unlike previous analyses of this system, which assumed $eV > \phi$, our analysis is valid in the low bias regime, $eV < \phi$, as well. The unconditional master equation is

given by Eq. (5) with $\Theta(-eV + \phi) > 0$, so it includes the term $\mathcal{D}[c_2^\dagger]\rho(t)$. In contrast to the high bias, in the low bias unravelling we assume that a jump c_2^\dagger is distinguishable from jumps c_1 and c_3 using current measurement (though the latter two are still mutually indistinguishable), since when the qubit is subject to a jump of type c_2^\dagger a lead electron hops from the drain to the source producing a current with opposite sign to the other two jump processes. The solution to the unconditional master equation in the low bias regime is

$$\begin{aligned} \rho_{ge}(t) &= e^{-(2\Gamma_d \cos^2(\theta) + \nu^2 \phi \sin^2(\theta))t} \rho_{ge}(0), \\ \rho_{gg}(t) &= 1 - e^{-2\nu^2 \phi \sin^2(\theta)t} (1 - \rho_{gg}(0)), \end{aligned} \quad (11)$$

Note that at $\phi = eV$, Eqs. (10) and (11) agree.

The unconditional, steady-state probability of the qubit to be found in its ground state is given by $\rho_{gg}(\infty) = \langle g | \rho(\infty) | g \rangle$. From Eqs. (10) and (11), we see that $\rho_{gg}(\infty)$ depends on the regime in which the PC is operating

$$\rho_{gg}(\infty) = \begin{cases} \frac{eV + \phi}{2eV} & \text{if } \phi < eV \\ 1 & \text{if } \phi > eV \end{cases} \quad (12)$$

In contrast, if the qubit were in thermal equilibrium with a heat bath at temperature T , then the ground state occupation would be $\rho_{gg}^{\text{therm}}(\infty) = e^{\phi/T} / (1 + e^{\phi/T})$. Figure 2 shows these two ground state probabilities for the case where $T = eV/2$. There is an evident analogy between the PC bias voltage and an external heat bath. We note that the leads are each nominally at zero temperature, that is $f_{S,D}(\omega) = H(\mu_{S,D} - \omega)$.

This correspondence between the PC bias and a temperature agrees with the predictions of [11] which shows that a PC induces effective thermal fluctuations in the position of a nearby harmonic oscillator. Even in the limit $eV \rightarrow 0$, the PC still causes the qubit to relax, and we can see from Eq. (11) that in this regime, the energy relaxation time is $\tau_r^{-1} = 2\nu^2 \phi \sin^2(\theta)$, and the dephasing time is $\tau_d^{-1} = 2\Gamma_d \cos^2(\theta) + \nu^2 \phi \sin^2(\theta)$. When $eV = 0$, $\tau_d = 2\tau_r$, indicating that the dephasing is solely due to energy relaxation. This zero-bias, zero-temperature dephasing arising from relaxation is the solid state analogue of the optical decay of an atom in a vacuum. This is practically significant, as it means the PC cannot be turned off merely by making $eV = 0$.

Another time scale of practical importance is the measurement time, τ_m . Following [7], we calculate the initial rate at which the z -component of the Bloch vector increases, starting from the symmetric state $|\psi(0)\rangle = (|l\rangle + |r\rangle)/\sqrt{2}$. This is given by $E[dz_c^{(e)}(t)^2] \equiv E[\text{Tr}\{\sigma_z^{(e)} d\rho_c(t)\}^2] = \tau_m^{-1} dt$. We find $\tau_m^{-1} = 2\Gamma_d \cos^2(\theta) + O(\nu^3 \phi)$. Also, $\tau_m \geq \tau_d$, with equality only when $\theta = 0$, indicating that the detector is inefficient unless the qubit energy eigenstates are localised.

We now calculate current power spectra, using the two-time correlation function $G(\tau) = E[I(t+\tau)I(t)] - E[I(t+)$

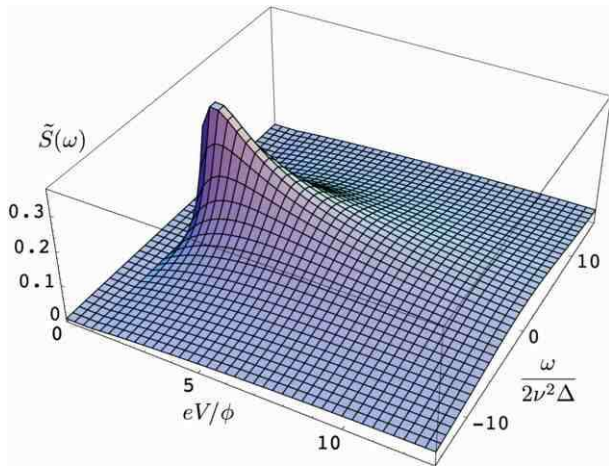


FIG. 3: Current power spectra, $\tilde{S}(\omega) = \frac{2\phi\nu^2\Delta^2}{\epsilon^2\delta I_{\text{loc}}^2}(S(\omega) - S(\infty))$. For a given ν the plot is valid in the region $eV/\phi \ll 1/\nu^2$. Note the sharp change in the power spectrum at $\phi = eV$.

$\tau)E[I(t)]$, where $E[\dots]$ is the classical expectation. The calculation follows the appendix of [6], using Eq. (9). The power spectrum is then $S(\omega) = 2 \int_{-\infty}^{\infty} d\tau G(\tau)e^{-i\omega\tau}$. To compute the steady state correlation function, we take the limit $t \rightarrow \infty$, so $\rho(t) \rightarrow \rho_{\infty}$. Keeping only the lowest order terms in a series expansion in δI_{loc} , then taking the Fourier transform gives

$$S_{\text{hb}}(\omega) = S_0 + \delta I_{\text{loc}} \frac{\phi}{V} \cos(\theta) + \frac{\delta I_{\text{loc}}^2 \sin^2(2\theta) \Gamma_d \frac{(eV)^2 - \phi^2}{(eV)^2}}{4(\Gamma_d^2 \sin^4(\theta) + \omega^2)}$$

where $S_0 = 2e\bar{I}_{\text{loc}}$.

In the low bias regime, there are two distinguishable jump processes that occur: source to drain electron tunnelling (both elastic and inelastic), and the reverse (inelastic only). Therefore the current is related to the number of jumps in the time interval by $i dt = e(dN^+(t) - dN^-(t))$, where $dN^{+(-)}(t)$ counts the number of source-to-drain (drain-to-source) tunnelling events in the time interval $(t, t + dt)$. We find the steady-state, low bias current correlation function is just due to elastic tunnelling through the PC,

$$S_{\text{lb}}(\omega) = S_0 + e \delta I_{\text{loc}} \cos(\theta) + O(\delta I_{\text{loc}}^2).$$

Figure 3 shows power spectra for different eV/ϕ . These power spectra are notably different from those predicted elsewhere in the sub-Zeno limit [6, 9, 10, 13], with the absence of coherent oscillations at $\omega = \pm\phi$.

In conclusion, we have shown that, in the sub-Zeno regime, inelastic tunnelling processes through a PC suppress coherent oscillations in the detector output, contrary to previous claims. Projective measurements in the localised basis are still possible as long as the energy eigenstates themselves are localised. If the eigenstates

are not localised, inelastic jumps reduce the detector efficiency. These inelastic jumps also generate pseudo-thermal fluctuations in the qubit. This is true even when the source-drain bias is zero, leading to decoherence of the qubit when the detector is nominally off. This is of practical significance in a QIP, since it means the detector will act as a source of decoherence.

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